

Precise Measurements of Primordial Power Spectrum with 21 cm Fluctuations

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Abstract

We discuss the issue of how precisely we can measure the primordial power spectrum by using future observations of 21 cm fluctuations and cosmic microwave background (CMB). For this purpose, we investigate projected constraints on the quantities characterizing primordial power spectrum: the spectral index n_s , its running α_s and even its higher order running β_s . We show that future 21 cm observations in combinations with CMB would accurately measure above mentioned observables of primordial power spectrum. We also discuss its implications to some explicit inflationary models.

1 Introduction

It is now widely believed that the Universe experienced an extremely rapid expanding phase, the inflation, in the early stage of its history, which is assumed to be driven by the vacuum energy of a scalar field, called inflaton. In addition that the inflaton gives an inflationary expanding era, it can also provide the source of primordial density fluctuations, whose nature can be utilized to differentiate inflationary models. Specifically, the spectral index n_s , describing the scale dependence of the primordial power spectrum, and the tensor-to-scalar ratio r , characterizing the amplitude of the gravity waves, are usually discussed, which can be measured/constrained by cosmological observations such as cosmic microwave background (CMB), large scale structure and so on. In particular, in the near future, several experiments are expected to measure/constrain the tensor-to-scalar ratio with high sensitivity, which may enable us to grasp invaluable hints to understand the inflationary model realized in the nature.

However, on the other hand, there exist inflation models which predict too small tensor-to-scalar ratio to be detected, at least, in near future experiments. Furthermore, primordial density fluctuations are not necessarily generated from the inflaton, but they can be produced by a light scalar field other than the inflaton, such as in the curvaton model [1–3], modulated reheating scenario [4,5] and so on. In particular, models of these kind have been attracting attention since they can predict large values of f_{NL} , a measure of bispectrum which characterizes non-Gaussian nature of density fluctuations^{#1}. In these models, the tensor-to-scalar ratio is generally very small, and thus one may not be able to detect any gravity wave signal if such a model is the correct one for generating density fluctuations. In such a case, we may need to pursue another quantity which is not related to the gravity waves. As such a quantity, the spectral index n_s is widely discussed. However, there are many inflationary models which can predict almost the same value of n_s , thus we need something more.

In light of these considerations, it would be desirable to have more observables, in particular, which are irrelevant to the tensor mode. Needless to say, such a quantity would also be helpful even if we can have information from the gravity waves as well, and give complementary information. As such an observable, we consider the running of the spectral index, α_s and even a higher order running or “the running of the running” β_s . The former quantity, α_s , has already been explored by many authors. However, the latter one, β_s , has not been investigated much in the literature^{#2}, on which we especially focus in this paper. In general, higher order runnings are expected to be very small compared to the spectral index $n_s - 1$, thus it seems very difficult to actually measure such a quantity. However, as we will show in this paper, future experiments of 21 cm fluctuation can give precise measurements of α_s and β_s , which would be very useful to differentiate inflationary models.

^{#1} Although the standard inflation models generally predict $f_{\text{NL}} \ll \mathcal{O}(1)$, current observations of CMB and large scale structure give the constraints on f_{NL} for the local type as $-3 < f_{\text{NL}} < 77$ (95% C.L.) from WMAP9 [6] and $25 < f_{\text{NL}} < 117$ (95% C.L.) from NRAO VLA Sky Survey [8].

^{#2} See Refs. [9, 10] for examples of such works.

In fact, the issue of probing the primordial power spectrum with 21 cm experiments was discussed in Refs. [11, 12]. In this paper, we further extend the discussion including the running of the running β_s and study its expected constraints from future observations of 21 cm fluctuations in combination with CMB. In addition to observational constraints, we also study the runnings α_s and β_s as well as n_s in some explicit models and discuss its testability and in what cases the information from α_s and β_s will be useful to differentiate inflationary models.

This paper is organized into four sections as follows. In the next section, we give a formalism to discuss the power spectrum of primordial density fluctuations, paying particular attention to its scale dependence such as n_s, α_s and β_s . We also give the predictions for these variables in some explicit models and discuss in what models/cases the information of the runnings are useful. In Section 3, we describe our method to derive expected constraints from future observations of 21 cm fluctuations and CMB and present out results. The final section is devoted to summary of this paper.

2 Inflationary parameters: Formalism

As mentioned in the introduction, the purpose of this paper is to investigate what kind of information can be obtained by considering the scale dependence of primordial curvature perturbations including “higher order” ones. Before going to study expected constraints, here in this section, we give a formalism and formulas to discuss primordial curvature perturbations and its scale dependence. We also study some explicit inflationary models. Since primordial curvature perturbation can also be generated from a light scalar field other than the inflaton such as in the curvaton, modulated reheating scenarios and so on, we give the formulas for such cases as well. But first we start with a case of the standard inflation model.

2.1 Standard inflation case

For the standard inflation case, the primordial curvature power spectrum is given, in terms of the δN formalism, by

$$\mathcal{P}_\zeta(k) = \left(\frac{\partial N}{\partial \phi_*} \right)^2 \mathcal{P}_{\delta\phi}(k), \quad (1)$$

where $\mathcal{P}_{\delta\phi} = (H_*/2\pi)^2 (1 + 2(3C - 1)\epsilon - 2C\eta)$ is the power spectrum for fluctuations of the inflaton field ϕ [13, 14]. For the standard inflation case, $\partial N / \partial \phi_* = -H_*/\dot{\phi}_*$ with a dot representing derivative with respect to time. The subscript “*” indicates that the quantity is evaluated at the time of horizon exit $k = a_* H_*$ and $C = 2 - \ln 2 - b \simeq 0.73$ with b being

the Euler-Mascheroni constant. ϵ and η are slow-roll parameters which are defined as

$$\epsilon = \frac{1}{2} M_{\text{pl}}^2 \left(\frac{V_\phi}{V} \right)^2, \quad (2)$$

$$\eta = M_{\text{pl}}^2 \frac{V_{\phi\phi}}{V}, \quad (3)$$

where V is the potential for the inflaton and a subscript ϕ represents a derivative with respect to the inflaton field. For later use, here we also define higher order slow-roll parameters:

$$\xi^{(2)} = M_{\text{pl}}^4 \frac{V_\phi V_{\phi\phi\phi}}{V^2}, \quad (4)$$

$$\sigma^{(3)} = M_{\text{pl}}^6 \frac{(V_\phi)^2 V_{\phi\phi\phi\phi}}{V^3}, \quad (5)$$

where we have put the superscripts (2) and (3) to remind that $\xi^{(2)}$ and $\sigma^{(3)}$ are second and third order slow-roll quantities, respectively.

We expand the power spectrum in terms of the logarithm of the wave number up to the 3rd order as

$$\begin{aligned} \mathcal{P}_\zeta(k) &= \mathcal{P}_\zeta(k_{\text{ref}}) \exp \left[(n_s - 1) \ln \left(\frac{k}{k_{\text{ref}}} \right) + \frac{1}{2} \alpha_s \ln^2 \left(\frac{k}{k_{\text{ref}}} \right) + \frac{1}{3!} \beta_s \ln^3 \left(\frac{k}{k_{\text{ref}}} \right) \right] \\ &= \mathcal{P}_\zeta(k_{\text{ref}}) \left(\frac{k}{k_{\text{ref}}} \right)^{n_s - 1 + \frac{1}{2} \alpha_s \ln(k/k_{\text{ref}}) + \frac{1}{6} \beta_s \ln^2(k/k_{\text{ref}})}, \end{aligned} \quad (6)$$

where we have defined

$$\alpha_s = \frac{dn_s}{d \ln k}, \quad (7)$$

$$\beta_s = \frac{d\alpha_s}{d \ln k}. \quad (8)$$

α_s is usually called “running” of n_s . Thus in this sense, β_s may be called as “the running of the running.”

By using the slow-roll parameters, n_s , α_s and β_s are written as

$$n_s - 1 = -6\epsilon + 2\eta + \left(-\frac{10}{3} + 24C \right) \epsilon^2 + \frac{2}{3} \eta^2 - (2 + 16C) \epsilon \eta + \left(\frac{2}{3} + 2C \right) \xi^{(2)}, \quad (9)$$

$$\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\xi^{(2)}, \quad (10)$$

$$\beta_s = -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(3)}. \quad (11)$$

2.2 Case with a light scalar field

When the primordial curvature perturbation originates to fluctuations of a light scalar field σ other than the inflaton, such as in the case of the curvaton and modulated reheating models, the expressions for the spectral index and its runnings are different from those in the standard inflation case. Here we assume that the energy density of such a light scalar field is subdominant during inflation, and hence it does not affect the inflationary dynamics. To make our discussion general, we consider the case where the inflaton and a light scalar field can be both responsible for density perturbations^{#3}. Cases with the standard inflation or a light scalar field being purely responsible for density perturbations are given as limiting cases.

Denoting a light field as σ and assuming that the inflaton and the σ field are uncorrelated, the total curvature perturbation can be written as

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta^{(\phi)}(k) + \mathcal{P}_\zeta^{(\sigma)}(k), \quad (12)$$

where $\mathcal{P}_\zeta^{(\phi)}$ and $\mathcal{P}_\zeta^{(\sigma)}$ respectively represent the contribution from the inflaton and σ fields. The inflaton part $\mathcal{P}_\zeta^{(\phi)}$ is actually the one given in Eq. (1). For the contribution from a light scalar field, $\mathcal{P}_\zeta^{(\sigma)}$ is given by

$$\mathcal{P}_\zeta^{(\sigma)}(k) = \left(\frac{\partial N}{\partial \sigma_*} \right)^2 \mathcal{P}_{\delta\sigma}(k), \quad (13)$$

where $\mathcal{P}_{\delta\sigma} = (H_*/2\pi)^2 (1 - 2(1 - C)\epsilon - 2C\eta_\sigma)$ is the power spectrum for fluctuations of a light scalar field σ [14]. Here the slow-roll parameter η_σ for σ field is defined similarly to those for the inflaton case as

$$\eta_\sigma = \frac{U_{\sigma\sigma}}{3H_*^2}, \quad (14)$$

where we assume that the scalar potential is given by the sum of those for ϕ and σ as $V(\phi) + U(\sigma)$. For later use, here we also define other higher order slow-roll parameters as

$$\xi_\sigma^{(2)} = \frac{U_\sigma U_{\sigma\sigma\sigma}}{(3H_*^2)^2}, \quad (15)$$

$$\sigma_\sigma^{(3)} = \frac{(U_\sigma)^2 U_{\sigma\sigma\sigma}}{(3H_*^2)^3}, \quad (16)$$

where a subscript σ denotes the derivative with respect to σ field.

^{#3} Models of this kind are called “mixed” model. Such models have been investigated in the context of the curvaton model [15–19] and modulated reheating scenario [20].

The spectral index n_s and its runnings α_s and β_s in this case are given as follows:

$$\begin{aligned}
n_s - 1 &= \Xi_\phi \left[-6\epsilon + 2\eta + \left(-\frac{10}{3} + 24C \right) \epsilon^2 + \frac{2}{3} \eta^2 - (2 + 16C) \epsilon \eta + \left(\frac{2}{3} + 2C \right) \xi^{(2)} \right] \\
&\quad + \Xi_\sigma \left[-2\epsilon + 2\eta_\sigma + \left(-\frac{22}{3} + 8C \right) \epsilon^2 + \frac{2}{3} \eta_\sigma^2 + \left(\frac{8}{3} - 4C \right) \epsilon \eta + \left(\frac{2}{3} - 4C \right) \epsilon \eta_\sigma \right. \\
&\quad \left. + \frac{4}{3} \xi^{(2)} + 2C \xi_\sigma^{(2)} \right], \tag{17}
\end{aligned}$$

$$\begin{aligned}
\alpha_s &= \Xi_\phi (-24\epsilon^2 + 16\epsilon\eta - 2\xi_\phi^{(2)}) + \Xi_\sigma (-8\epsilon^2 + 4\epsilon\eta_\phi + 4\epsilon\eta_\sigma - 2\xi_\sigma^{(2)}) \\
&\quad - 4\Xi_\phi \Xi_\sigma (4\epsilon + 2\eta_\phi - 2\eta_\sigma)^2, \tag{18}
\end{aligned}$$

$$\begin{aligned}
\beta_s &= \Xi_\phi (-192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(3)}) \\
&\quad \Xi_\sigma (-64\epsilon^3 + 56\epsilon^2\eta_\phi - 8\epsilon\eta_\phi^2 + 24\epsilon^2\eta_\sigma - 8\epsilon\eta_\phi\eta_\sigma - 12\epsilon\xi_\phi^{(2)} - 4\epsilon\xi_\sigma^{(2)} + 2\eta_\sigma\xi_\sigma^{(2)} + 2\sigma_\sigma^{(3)}), \\
&\quad 12\Xi_\phi \Xi_\sigma (2\epsilon - \eta + \eta_\sigma)(8\epsilon^2 - 6\epsilon\eta_\phi + 2\epsilon\eta_\sigma + \xi^{(2)} - \xi_\sigma^{(2)}) \\
&\quad + 8\Xi_\phi \Xi_\sigma (\Xi_\phi - \Xi_\sigma)(2\epsilon - \eta + \eta_\sigma)^3, \tag{19}
\end{aligned}$$

where we have defined the fraction of $\mathcal{P}_\zeta^{(\phi)}$ and $\mathcal{P}_\zeta^{(\sigma)}$ in the total power spectrum, respectively, as

$$\Xi_\phi \equiv \frac{\mathcal{P}_\zeta^{(\phi)}}{\mathcal{P}_\zeta^{(\phi)} + \mathcal{P}_\zeta^{(\sigma)}}, \quad \Xi_\sigma \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)} + \mathcal{P}_\zeta^{(\sigma)}}. \tag{20}$$

By definition, the sum of these quantities is unity, $\Xi_\phi + \Xi_\sigma = 1$.

2.3 Some example

Now we discuss some explicit models and their predictions for the spectral index n_s and the runnings α_s and β_s . To provide an example of in what cases the information from the runnings is useful, here we consider models with small tensor-to-scalar ratio, in other words, the information from the (scalar-mode) power spectrum only available. We will argue below that, in some cases, “running of the running” can be very useful to discriminate inflationary models. For descriptive purpose, we consider three models and discuss their predictions on n_s , α_s and β_s .

Model I : Curvaton with quartic chaotic inflation model

In this model, we assume that the curvaton field is totally responsible for the cosmic density perturbations. However, even in the curvaton model, the inflationary expansion is driven by the inflaton field and we need to specify the inflation model,

or the inflaton potential, to compute the power spectrum. For the inflation model, we adopt the chaotic inflation model with a quartic potential:

$$V(\phi) = \lambda\phi^4, \quad (21)$$

where λ is a coupling parameter. In fact, the quartic chaotic inflation is already excluded by current cosmological observations [7] when the inflaton is completely responsible for density perturbations. However, when the curvaton generates primordial density fluctuations, it is still viable, and even more worth mentioning, such an inflation model is preferable for this case [17, 19] since it can give the spectral index of $n_s \sim 0.96$, which is close to the central value from observational constraints. We should also note here that, although the curvaton model has been attracting attention because it can generate large non-Gaussianity, but it is not necessarily large: the curvaton can also predict small values of f_{NL} in some parameter range. Thus, non-Gaussianity may not give useful information in such a case even for this type of model.

For the curvaton sector, we assume that its mass is very small so that we can set $\eta_\sigma = 0$ in the calculation of n_s and so on. Furthermore, as mentioned above, the curvaton is assumed to be totally responsible for the primordial power spectrum, and hence we also set $\Xi_\sigma = 1$ ($\Xi_\phi = 0$). To realize this situation, fluctuations from the inflaton should satisfy $P_\zeta^{(\phi)} \ll 2.4 \times 10^{-9}$, which means that the inflation scale is low and the tensor-to-scalar ratio becomes very small.

In Fig. 1, we plot the values of n_s , α_s and β_s in this model as well as those for Model II which will be discussed in the following. In the figure, the predictions are shown in the n_s - α_s (left panel) and the α_s - β_s (right panel) planes. Here the number of e -folds at the horizon exit is varied as $40 \leq N_e \leq 60$.

Model II :Mutated hybrid inflation model

For this model, we assume that the inflaton is fully responsible for density perturbations and the inflaton potential has the following form:

$$V(\phi) = V_0 \left[1 - \left(\frac{\mu}{\phi} \right)^q + \cdots \right], \quad (22)$$

which is called mutated hybrid inflation model [21] and q is assumed to be a positive integer. μ is some energy scale which characterizes the model.

In Fig. 1, we show the predictions for n_s , α_s and β_s in this model along with those for Model I. For illustration purposes, here we take $q = 8$ and $\mu = M_{\text{pl}}$. In the figure, N_e is varied in the range $N_e = 40 - 60$ to plot the lines. We note that, in both models, the tensor-to-scalar ratio is small as $r < 10^{-3}$. By looking at the prediction

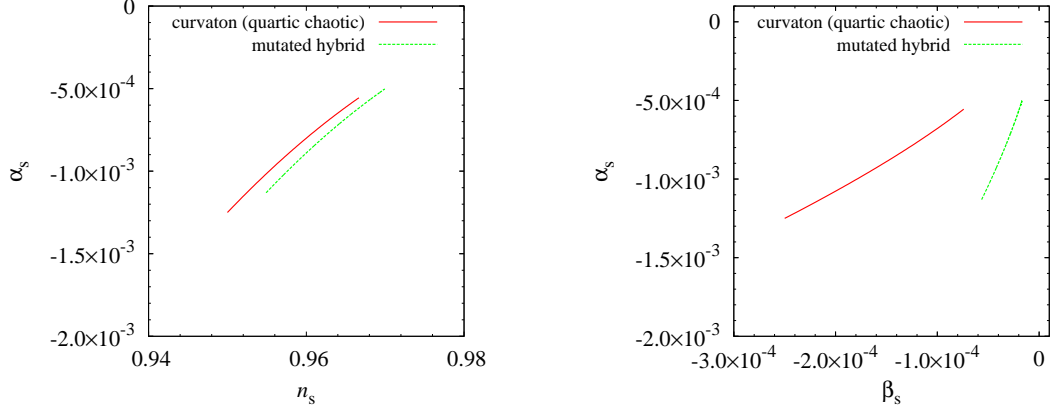


Figure 1: Predictions in the n_s - α_s (left) and the α_s - β_s (right) planes from curvaton model with chaotic inflation ($V(\phi) = \lambda\phi^4$) and mutated hybrid model with $q = 4$ and $\mu = M_{\text{pl}}$. The number of e -folds is varied as $40 \leq N_e \leq 60$ in this figure.

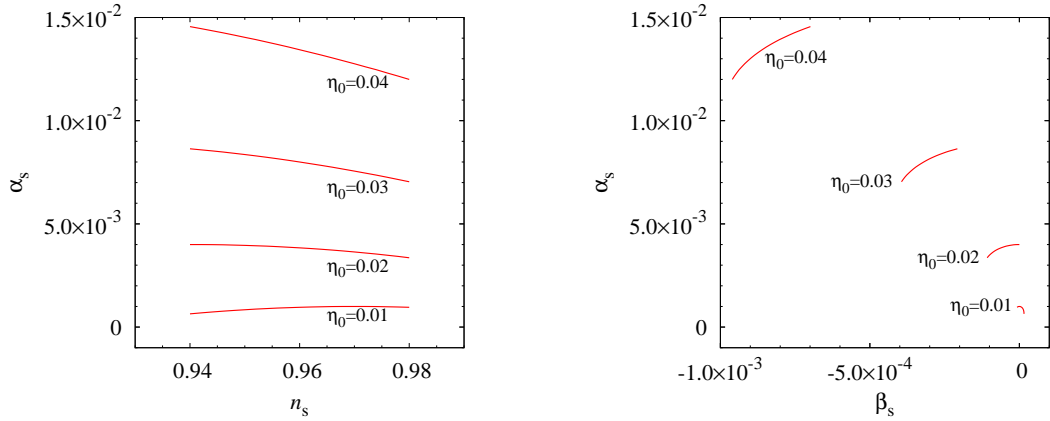


Figure 2: Predictions in the n_s - α_s (left) and the β_s - α_s (right) planes from Type III hilltop inflation models. From the top to bottom, η_0 is taken to be 0.04 , 0.03 , 0.02 , and 0.01 .

in the n_s - α_s plane, one can see that these models give very similar predictions for n_s and α_s . However, if we look at the α_s - β_s plane, their values seem separated compared to those in n_s - α_s plane, which shows an idea that a higher order running β_s can give useful information to differentiate inflationary models. To state this more quantitatively, we need to know attainable constraints on these observables from future cosmological observations, which is the topic in the next section.

Model III : Type-III hilltop inflation models

As a last example, we discuss a model in which both $|\alpha_s|$ and $|\beta_s|$ can be large. In so-called Type-III hilltop inflation models [22, 23] (see also Refs. [24–26] and references therein), we expect a large running and a large running of running towards small scales. The potential shape is represented by

$$V(\phi) = V_0 \left(1 + \frac{1}{2} \eta_0 \frac{\phi^2}{M_{\text{pl}}^2} \right) - \lambda \frac{\phi^q}{M_{\text{pl}}^{q-4}}. \quad (23)$$

The field ϕ rolls from the top of the hill towards the origin during the inflationary epoch. Intermediately, the inflation ends by a water-fall mechanism. In Fig. 2, we plot the predictions in the n_s - α_s (left) and the β_s - α_s (right) planes, in case of $q = 6$ and $V_0^{1/4}/M_{\text{pl}} = 1.6 \times 10^{-7}$. Because the tensor-to-scalar ratio is tiny with an order of $r \sim 2 \times 10^{-20}$ in this model, it would be difficult to detect it. However, with future precise measurements of α_s and β_s , we will be able to discriminate this model from others.

3 Analysis

Now we discuss expected observational constraints on the quantities such as the spectral index n_s , its runnings α_s and β_s from future cosmological observations by using Fisher matrix formalism. For this purpose, we investigate a joint constraint from 21cm fluctuations and CMB. We first briefly describe the formalism in each below, then present our results on the future constraints.

3.1 21cm power spectrum

Here we briefly sketch and summarize how we can constrain primordial power spectrum by using 21 cm observations. For detailed descriptions, we refer the readers to, e.g., Refs. [11, 27–30].

The 21 cm line from the hyperfine transition of neutral hydrogen can be observed as the differential brightness temperature relative to the CMB temperature T_{cmb} :

$$T_b(\mathbf{x}) = \frac{3c^3 h A_{10} n_H(\mathbf{x}) (T_s(\mathbf{x}) - T_{\text{cmb}})}{32\pi k_B \nu_{21}^2 T_s(\mathbf{x}) (1+z)^2 (dv_{\parallel}/dr)}, \quad (24)$$

where $A_{10} \simeq 2.85 \times 10^{-15} \text{s}^{-1}$ is the spontaneous decay rate of 21 cm transition and ν_{21} is the 21 cm line (rest) frequency. $T_s(\mathbf{x})$ is the spin temperature, which is defined by $n_1/n_0 = 3 \exp(-T_{21}/T_S)$ with n_0 and n_1 being respectively the number densities of singlet $1_0 S_{1/2}$ and triplet $1_1 S_{1/2}$ states of neutral hydrogen atom. Here $T_{21} = hc/k_B \lambda_{21}$ is the temperature corresponding to 21 cm line with λ_{21} being its wavelength. dv_{\parallel}/dr is the gradient of the velocity along the line of sight.

Now we are going to consider fluctuations of $T_b(\mathbf{x})$. By expanding the hydrogen number density n_H and the ionization fraction x_i (we also sometimes use $x_H = 1 - x_i$: the fraction of neutral hydrogen) as $n_H(\mathbf{x}) = \bar{n}_H(1 + \delta(\mathbf{x}))$ and $x_i(\mathbf{x}) = \bar{x}_i(1 + \delta_x(\mathbf{x}))$, we can rewrite Eq. (24) as

$$T_b(\mathbf{x}) = \bar{T}_b (1 - \bar{x}_i(1 + \delta_x(\mathbf{x})) (1 + \delta(\mathbf{x})) \left(1 - \frac{1}{Ha} \frac{dv_r}{dr}\right), \quad (25)$$

where we have assumed that $T_s \gg T_{\text{cmb}}$ since we focus on the epoch of reionization during which this condition is well satisfied. \bar{T}_b is the spatially averaged brightness temperature at redshift z and given by

$$\bar{T}_b \simeq 27 \text{ mK} \left(\frac{\Omega_b h^2}{0.023} \right) \left(\frac{0.15}{\Omega_m h^2} \frac{1+z}{10} \right)^{1/2}, \quad (26)$$

and dv_r/dr is the peculiar velocity along the line of sight.

By denoting the fluctuation in T_b as $\delta T_b(\mathbf{x}) = T_b(\mathbf{x}) - \bar{x}_H \bar{T}_b$, the 21 cm power spectrum $P_{21}(\mathbf{k})$ in the k -space is defined by

$$\langle \delta T_b^*(\mathbf{k}) \delta T_b(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P_{21}(\mathbf{k}). \quad (27)$$

z	\bar{x}_H	b_{xx}^2	R_{xx} [Mpc]	α_{xx}	γ_{xx}	$b_{x\delta}^2$	$R_{x\delta}$ [Mpc]	$\alpha_{x\delta}$
9.2	0.9	0.208	1.24	-1.63	0.38	0.45	0.56	-0.4
8.0	0.7	2.12	1.63	-0.1	1.35	1.47	0.62	0.46
7.5	0.5	9.9	1.3	1.6	2.3	3.1	0.58	2.0
7.0	0.3	77.0	3.0	4.5	2.05	8.2	0.143	28.0

Table 1: Fiducial values for the parameters in $\mathcal{P}_{xx}(k)$ and $\mathcal{P}_{x\delta}(k)$ (See Eqs. (32) and (33)) [11].

By treating the peculiar velocity $\delta_v \equiv (dv_r/dr)(1/aH)$ as a perturbation and using that its Fourier transform can be given by $\delta_v(\mathbf{k}) = -\mu^2\delta(\mathbf{k})$ with $\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$ being the cosine of the angle between the wave vector and the line of sight, the power spectrum can be written as

$$P_{21}(\mathbf{k}) = P_{\mu^0}(k) + \mu^2 P_{\mu^2}(k) + \mu^4 P_{\mu^4}(k), \quad (28)$$

where $k = |\mathbf{k}|$ and

$$P_{\mu^0} = \mathcal{P}_{\delta\delta} - 2\mathcal{P}_{x\delta} + \mathcal{P}_{xx}, \quad (29)$$

$$P_{\mu^2} = 2(\mathcal{P}_{\delta\delta} - \mathcal{P}_{x\delta}), \quad (30)$$

$$P_{\mu^4} = \mathcal{P}_{\delta\delta}. \quad (31)$$

Here $\mathcal{P}_{\delta\delta} = \bar{T}_b^2 \bar{x}_H^2 P_{\delta\delta}$, $\mathcal{P}_{x\delta} = \bar{T}_b^2 \bar{x}_i \bar{x}_H P_{x\delta}$ and $\mathcal{P}_{xx} = \bar{T}_b^2 \bar{x}_i^2 P_{xx}$ and $P_{\delta\delta}$, $P_{x\delta}$ and P_{xx} are the power spectra defined in the same manner as Eq. (27) for δ and δ_x . Since δ represents fluctuations in the hydrogen number density, $P_{\delta\delta}$ traces that of matter which includes the information on primordial power spectrum. $P_{x\delta}$ and P_{xx} can be neglected if we consider the era when the intergalactic medium (IGM) is completely neutral. However, after the reionization starts, in which we are interested, these two spectra contribute significantly. Although a rigorous evaluation of these power spectra may need some numerical simulations, here we adopt the treatment given in Ref. [11], where $\mathcal{P}_{x\delta}$ and \mathcal{P}_{xx} are assumed to have a specific form to match radiative transfer simulations of Refs. [31, 32]. Their explicit forms are the followings:

$$\mathcal{P}_{xx}(k) = b_{xx}^2 [1 + \alpha_{xx}(kR_{xx}) + (kR_{xx})^2]^{-\gamma_{xx}/2} \mathcal{P}_{\delta\delta}(k), \quad (32)$$

$$\mathcal{P}_{x\delta}(k) = b_{x\delta}^2 e^{-\alpha_{x\delta}(kR_{x\delta}) - (kR_{x\delta})^2} \mathcal{P}_{\delta\delta}(k), \quad (33)$$

where b_{xx} , $b_{x\delta}$, α_{xx} , γ_{xx} and $\alpha_{x\delta}$ are parameters which characterize the amplitudes and the shapes of the spectra. R_{xx} and $R_{x\delta}$ correspond to the effective size of the ionized bubbles. The values of these parameters we adopt in the analysis are listed in Table 1.

Now we are in the position to discuss the Fisher matrix for 21 cm observations. First of all, we note that experiments of 21 cm radiation do not directly measure the wave number \mathbf{k} nor the power spectrum in k -space $P_{21}(\mathbf{k})$. Instead, an experiment measures the angular

location on the sky and the frequency which can be specified by the vector

$$\mathbf{\Theta} = \theta_x \hat{e}_x + \theta_y \hat{e}_y + \Delta f \hat{e}_z (\equiv \mathbf{\Theta}_\perp + \Delta f \hat{e}_z). \quad (34)$$

Here the frequency is represented by its difference from the central redshift z of a given redshift bin. Then we can define the Fourier dual of $\mathbf{\Theta}$ as

$$\mathbf{u} \equiv u_x \hat{e}_x + u_y \hat{e}_y + u_\parallel \hat{e}_z (\equiv \mathbf{u}_\perp + u_\parallel \hat{e}_z). \quad (35)$$

Notice that, since u_\parallel is the Fourier dual of Δf , it has the units of time. Assuming that the sky is flat^{#4}, we can linearize the relation \mathbf{r} and $\mathbf{\Theta}$. Denoting the vector perpendicular to the line of sight as \mathbf{r}_\perp , we have the relations

$$\mathbf{\Theta}_\perp = \mathbf{r}_\perp / d_A(z_*), \quad \Delta f = \Delta r_\parallel / y(z_*) \quad (36)$$

where $d_A(z_*)$ is the comoving angular diameter distance and $y(z) = \lambda_{21}(1+z)^2/H(z)$. Δr_\parallel is the comoving distance intervals corresponding to the frequency intervals Δf . Then, the relation between \mathbf{k} and \mathbf{u} can be written as

$$\mathbf{u}_\perp = d_A \mathbf{k}_\perp, \quad u_\parallel = y k_\parallel. \quad (37)$$

The power spectrum of δT_b in u -space can be defined in exactly the same manner as that for k -space by replacing \mathbf{k} with \mathbf{u} in Eq. (27). By using the relation between \mathbf{k} and \mathbf{u} , the power spectra in each space are connected as

$$P_{21}(\mathbf{u}) = \frac{1}{d_A(z)^2 y(z)} P_{21}(\mathbf{k}). \quad (38)$$

We use the u -space power spectrum in the following analysis.

With the power spectrum $P_{21}(\mathbf{u})$, the Fisher matrix is given by

$$F_{ij}^{21\text{cm}} = \sum_{\text{pixels}} \frac{1}{[\delta P_{21}(\mathbf{u})]^2} \left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_i} \right) \left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_j} \right), \quad (39)$$

where $\delta P_{21}(\mathbf{u})$ is the error in the power spectrum measurements for a pixel \mathbf{u} , and p_i represents cosmological parameters. To be conservative, when we differentiate $P_{21}(\mathbf{u})$ with respect to cosmological parameters, we fix $\mathcal{P}_{\delta\delta}(k)$ in Eqs. (32) and (33) so that constraints only come from the $\mathcal{P}_{\delta\delta}(k)$ terms in $P_{\mu^0}, P_{\mu^2}, P_{\mu^4}$. The error of the power spectrum $\delta P_{21}(\mathbf{u})$ comes from sample variance and experimental noise, and is given by

$$\delta P_{21}(\mathbf{u}) = \frac{P_{21}(\mathbf{u}) + P_N(u_\perp)}{N_c^{1/2}}. \quad (40)$$

^{#4} Even if we consider all-sky experiments, the flat-sky approximation can be valid as long as the data are analyzed in many small patches of the sky [11].

Experiment	N_{ant}	$A_e(z=8)$ [m ²]	L_{min} [m]	L_{max} [km]	FoV [deg ²]	z
SKA	7000	50	10	10	$\pi \ 8.6^2$	$6.8 - 10$
Omniscope	10^6	1	1	1	2.06×10^4	$6.8 - 10$

Table 2: Specifications for interferometers of 21 cm experiments adopted in the analysis. For Omniscope, we assume that the effective collecting area A_e and field of view are fixed.

The first term on the right hand side gives the one from the sample variance. $N_c = 2\pi k_\perp \Delta k_\perp \Delta k_\parallel V(z)/(2\pi)^3$ is the number of independent cells in an annulus summing over the azimuthal angle. Here $V(z) = d_A(z)^2 y(z) B \times \text{FoV}$ is the survey volume with B being the bandwidth and $\text{FoV} = \lambda^2/A_e$ is the field of view of the interferometer with $A_e \propto \lambda^2$ being the effective collecting area. The noise power spectrum, denoted as $P_N(u_\perp)$ in the above formula, is given by

$$P_N(u_\perp) = \left(\frac{\lambda(z) T_{\text{sys}}(z)}{A_e(z)} \right)^2 \frac{1}{t_0 n(u_\perp)}, \quad (41)$$

where $T_{\text{sys}} = 280[(1+z)/7.4]^{2.3}$ K is the system temperature [33] which is dominated by the sky temperature due to synchrotron radiation. t_0 is the total observation time and $n(u_\perp)$ is the number density of the baseline, which depends on actual realization of antenna distributions of each experiment.

In our analysis, we consider the redshift range $z = 6.75 - 10.05$, which we divide into 4 bins: $z = 6.75 - 7.25, 7.25 - 7.75, 7.75 - 8.25$ and $8.25 - 10.05$. For the wave number, we set $k_{\text{min}\parallel} = 2\pi/(yB)$ to avoid foreground contamination [29] and take $k_{\text{max}} = 2 \text{ Mpc}^{-1}$ in order not to be affected by nonlinear effect which becomes important on $k \geq k_{\text{max}}$.

To obtain the future cosmological constraints from 21 cm experiments, we consider SKA [34] and Omniscope [35] whose specifications are shown in Table 2. In order to calculate number density of baseline $n(u_\perp)$, we assume a realization of antenna distributions for both arrays in the same manner as Ref. [11]. For SKA, we take 16% of the total antennae distributed with a filled nucleus of radius 189 m, and these antennae are surrounded by 4% of the total antennae, which has an antenna density profile of an inverse square of the radius. These antennae are surrounded by a further 30% of the total antennae in a uniform density annulus of outer radius 5 km. We ignore the measurements from the sparse distribution of the remaining 50% of the total antennae that are outside the annulus. For Omniscope, which is a future square-kilometer collecting area array optimized for 21 cm tomography, we take all of antennae distributed with a filled nucleus. In addition, we assume an azimuthally symmetric distribution of the antenna in both arrays.

3.2 CMB

As mentioned previously, although 21 cm experiments have strong power to probe the primordial power spectrum, especially, on small scales, observations of CMB greatly help

experiment	frequency [GHz]	beam θ_{FWHM} [arcmin]	Δ_T [$\mu\text{K arcmin}$]	Δ_P [$\mu\text{K arcmin}$]
Planck	70	14	137	195
	100	9.5	64.6	104
	143	7.1	42.6	80.9
CMBpol	70	11	2.96	4.19
	100	8	2.29	3.24
	150	5	2.21	3.13

Table 3: Specifications for Planck and CMBpol adopted in the analysis. For CMBpol, we assumed the mid-cost (EPIC-2m) mission and only used three frequency bands (70 GHz, 100 GHz and 150 GHz) for a realistic foreground removal. In the both observations, $f_{\text{sky}} = 0.65$ is assumed.

to determine other cosmological parameters such as energy densities of dark matter, baryon and dark energy, and so on. To obtain the future constraints, we consider Planck [36] and CMBpol [37] whose specifications are summarized in Table 3.

The Fisher information matrix for CMB is given by [30, 38, 39]

$$F_{ij}^{(\text{CMB})} = \sum_l \frac{(2l+1)}{2} f_{\text{sky}} \text{Trace} \left[\mathbf{C}_l^{-1} \frac{\partial \mathbf{C}_l}{\partial p_i} \mathbf{C}_l^{-1} \frac{\partial \mathbf{C}_l}{\partial p_j} \right], \quad (42)$$

where \mathbf{C}_l is a covariance matrix of CMB, and p_i represents cosmological parameters. We take the maximum multipole to be $l_{\text{max}} = 3000$ in the analysis. The covariance matrix \mathbf{C}_l is expressed as

$$\mathbf{C}_l = \begin{pmatrix} C_l^{TT} + N_l^T & C_l^{TE} & C_l^{Td} \\ C_l^{TE} & C_l^{EE} + N_l^P & 0 \\ C_l^{Td} & 0 & C_l^{dd} + N_l^d \end{pmatrix}, \quad (43)$$

where C_l^X ($X = TT, EE, TE$) is an angular power spectrum of each unlensed CMB mode, C_l^{dd} is one of deflection angle of CMB due to weak gravitational lensing, C_l^{Td} is cross correlation between temperature and deflection angle, N_l^Y ($Y = T, P$) is a noise power spectrum of temperature or polarization, and N_l^d is one of deflection angle. The noise power spectrum N_l^Y is given by,

$$N_l^Y(\nu) = \Delta_Y^2 \exp [l(l+1)\sigma_b^2(\nu)], \quad (44)$$

for a single frequency band, where Δ_Y can be found in Table 3 and $\sigma_b(\nu) = \theta_{\text{FWHM}}/\sqrt{8 \ln 2}$ characterizes the width of the beam. When multiple frequency bands are used, where N_l^Y is given by the sum of its inverse as

$$(N_l^Y)^{-1} = \sum_{\nu_i} \frac{1}{N_l^Y(\nu_i)}. \quad (45)$$

In addition, we estimate the noise power spectrum of deflection angle N_l^d by assuming that we use lensing reconstruction with the quadratic estimator [40], and compute it by using a public code FUTURCMB [41], which adopts the estimator.

Equipped with these formalism, we can calculate the Fisher matrix to obtain projected constraints on the spectral index n_s and the runnings α_s and β_s , which will be presented in the next subsection.

3.3 Future constraints

Now we present our results for projected constraints on cosmological parameters, paying particular attention to n_s, α_s and β_s which characterize the scale-dependence of primordial curvature perturbations. In addition to these parameters, we also vary the standard cosmological parameters. Thus we explore an 8-dimensional parameter space: $\Omega_\Lambda, \Omega_b h^2, h, \tau, A_s, n_s, \alpha_s, \beta_s$, where Ω_Λ and Ω_b are energy densities of cosmological constant and baryon, respectively, h is the Hubble parameter in units of 100km/s/Mpc and τ is the reionization optical depth. In the analysis, we assume a flat universe, and fix the Helium abundance to be $Y_p = 0.25$. Neutrinos are treated as massless particles and its effective number is also fixed to be $N_{\text{eff}} = 3.046$. A_s, n_s, α_s and β_s are parameters which characterize the primordial power spectrum as already mentioned, and we normalize its amplitude as $A_s(k_{\text{ref}}) \equiv \mathcal{P}_\zeta(k_{\text{ref}})/(2.18 \times 10^{-9})$ [42, 43]. As a reference scale to parametrize these quantities, we take $k_{\text{ref}} = 0.05 \text{ Mpc}^{-1}$ in the most analysis presented below. However, we also discuss how this reference scale affects the determinations of n_s, α_s and β_s at the final part of this section. Then we set the fiducial values of these parameters (except α_s, β_s) near the best fit of the WMAP 7-year results [44], and $\alpha_s = 0, \beta_s = 0$, so that $(\Omega_\Lambda, \Omega_b h^2, h, \tau, A_s, n_s, \alpha_s, \beta_s) = (0.729, 0.02227, 0.703, 0.085, 1, 0.966, 0, 0)$.

The total Fisher matrix is given by

$$F_{ij} = F_{ij}^{(21\text{cm})} + F_{ij}^{(\text{CMB})}, \quad (46)$$

where i, j are subscript representing cosmological parameters. We report our results for several combinations of experiments. As already mentioned above, although 21 cm experiments enable us to probe the primordial power spectrum very precisely since they can measure fluctuations smaller scales compared to those of CMB, regarding the determinations of other cosmological parameters, CMB has more strong power. Since the scale-dependence of the power spectrum has degeneracies with other cosmological parameters, reducing the uncertainties of such parameters is also important to precisely measure n_s, α_s and β_s .

In Fig. 3, we show projected constraints in the n_s - α_s and α_s - β_s planes where 2σ limits are shown for the analysis from Planck, Planck+SKA, Planck+Omniscope and CMBpol, CMBpol+SKA, CMBpol+Omniscope. Uncertainties for each parameter n_s, α_s and β_s are summarized in Table 4. We note that the 1σ uncertainties are reported in the table. Although CMB experiments can already give a very precise measurement of n_s at better than $\mathcal{O}(1\%)$, when one includes the data from 21 cm fluctuations, especially with

	δn_s	$\delta \alpha_s$	$\delta \beta_s$
Planck	4.57×10^{-3}	7.84×10^{-3}	1.13×10^{-2}
Planck+ SKA	2.97×10^{-3}	4.06×10^{-3}	4.67×10^{-3}
Planck+ Omniscope	5.73×10^{-4}	1.06×10^{-3}	7.38×10^{-4}
CMBpol	2.12×10^{-3}	2.44×10^{-3}	4.53×10^{-3}
CMBpol+SKA	1.87×10^{-3}	2.33×10^{-3}	3.59×10^{-3}
CMBpol+Omniscopes	5.28×10^{-4}	9.90×10^{-4}	6.92×10^{-4}

Table 4: 1σ uncertainties for n_s, α_s and β_s from various data sets. We take $k_{\text{ref}} = 0.05 \text{ Mpc}^{-1}$ to derive these constraints.

Omniscopes, the precision improves by even one order of magnitude as seen from the figure and the table. The same also holds for the runnings α_s and β_s . In particular, for the case with CMBpol+Omniscopes, one can probe the runnings α_s and β_s with the precision down to $\mathcal{O}(10^{-4})$. As discussed in the previous section, some inflationary model predicts $\alpha_s = \mathcal{O}(10^{-3})$ and $\beta_s = \mathcal{O}(10^{-4})$, thus such models can be tested with future experiments of CMB and 21 cm fluctuations by using the runnings of the power spectrum.

Our results show that the scale-dependence of the primordial power spectrum can be well probed when one uses the data from 21 cm fluctuations, which is consistent with [11, 12], and further demonstrate that the information of “higher order” scale-dependence such as the running α_s and the running of the running β_s can also give significant information on models of primordial density fluctuations.

Finally we discuss how the choice of k_{ref} affects the determination of the parameters n_s, α_s and β_s . In Fig. 6, expected constraints on the n_s – α_s and α_s – β_s planes are shown for several values of k_{ref} for Planck+Omniscopes and CMBpol+Omniscopes. 1σ uncertainties for each parameter are summarized in Table 5. As seen from the figure, the choice of k_{ref} affects the uncertainties for n_s, α_s and β_s by a factor of a few and also changes the direction of the degeneracy. From the viewpoint of determining the scale dependence parameters, the optimal reference scale would be the one which gives uncorrelated constraints among the parameters. However, by looking at Fig. 6, we can see that the reference scale giving uncorrelated measures for the parameter sets (n_s, α_s) and (α_s, β_s) are different. In addition, even just considering CMB or 21 cm experiment alone, the optimal scale seems to also depend on the specification of the experiments. But a general tendency is that the optimal scales is around from 0.05 Mpc^{-1} to 0.1 Mpc^{-1} . Thus we have mainly used the reference scale $k_{\text{ref}} = 0.05 \text{ Mpc}^{-1}$ to show our results.

4 Conclusion

We have investigated how precisely one can measure the power spectrum of the curvature perturbation from future experiments of 21 cm fluctuations and CMB. In particular, we

k_{ref} [Mpc ⁻¹]	0.002	0.01	0.05	0.1	0.2	0.5
δn_s	3.74×10^{-3}	2.60×10^{-3}	5.28×10^{-4}	4.06×10^{-4}	4.73×10^{-4}	3.35×10^{-4}
$\delta \alpha_s$	1.47×10^{-3}	1.89×10^{-3}	9.90×10^{-4}	5.42×10^{-4}	2.64×10^{-4}	6.89×10^{-4}
$\delta \beta_s$	2.46×10^{-4}	6.05×10^{-4}	6.92×10^{-4}	6.94×10^{-4}	6.93×10^{-4}	6.85×10^{-4}

Table 5: Expected 1σ uncertainties of n_s , α_s and β_s from CMBpol+Omniscope for several values of k_{ref} .

have studied projected constraints on the parameters characterizing the scale-dependence of the power spectrum such as the spectral index n_s , its running α_s and the running of the running β_s . Although the former two parameters have been well explored in various context in the literature, the latter one, β_s , has not been studied much in connection with cosmological probes.

Although the gravity waves or the tensor mode can give significant information to the inflationary Universe once it is detected, there are many inflation models, such as small-field models, predicting too small tensor-to-scalar ratio. In addition, in models with a light scalar field such as the curvaton model, modulated reheating scenario and so on, which are of interest due to the possibilities of their giving large f_{NL} , the tensor-to-scalar ratio also tends to be very small. If one of these models is realized in the nature, it would be very difficult to detect the signature from the gravity waves. However, even in that case, “higher order” scale dependence of (scalar) curvature perturbations would help to probe the inflationary model, which we show quantitatively in this paper. We have discussed some explicit models where higher order running β_s would be very useful to differentiate models. Needless to say, even when the tensor modes are detected, the runnings can give extra valuable information on models of primordial fluctuations.

We have obtained expected constraints on such a higher order running β_s as well as n_s and α_s by using observations of 21 cm fluctuation, in combination with CMB. Since the power spectrum of 21 cm fluctuations can probe cosmic density fluctuations on smaller scales than those observed in CMB, one can obtain severe constraints even for the running of running parameter β_s . In particular, when one considers the combination of CMBpol+Omniscope, we can probe the running parameters with the precision of $\delta \alpha_s = \mathcal{O}(10^{-3})$ and $\delta \beta_s = \mathcal{O}(10^{-4})$, which would give useful information to discriminate inflationary models.

Although current cosmological observations are already so precise that some of inflationary models have been excluded, there are still many possibilities allowed and thus we need to go further to understand the early Universe more. In particular, to see the details and differentiate models well, it is preferable to have yet another observables other than commonly used one. As such a quantity, we considered a higher order running β_s in this paper, which can be well probed by future cosmological observations such as from 21 cm and CMB. In the view that cosmological data will be much more precise in the future, the

research along this line would provide us a lot of insight on the inflationary Universe and even the origin of the Universe.

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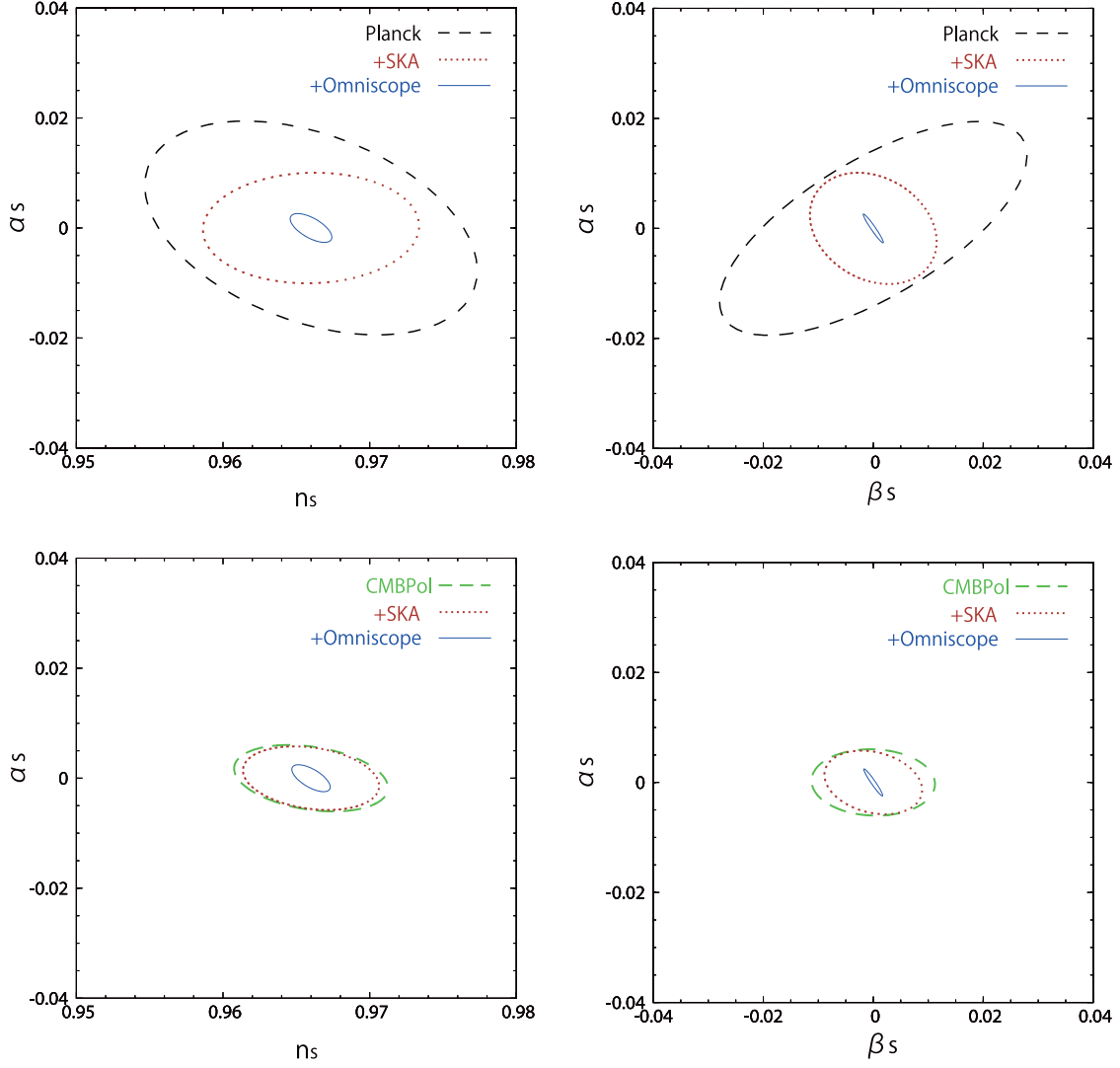


Figure 3: Projected constraints from Planck, Planck+SKA, Planck+Omniscope (top panels) and CMBpol, CMBpol+SKA, CMBpol+Omniscope (bottom panels) in the n_s - α_s (left) and α_s - β_s (right) planes.

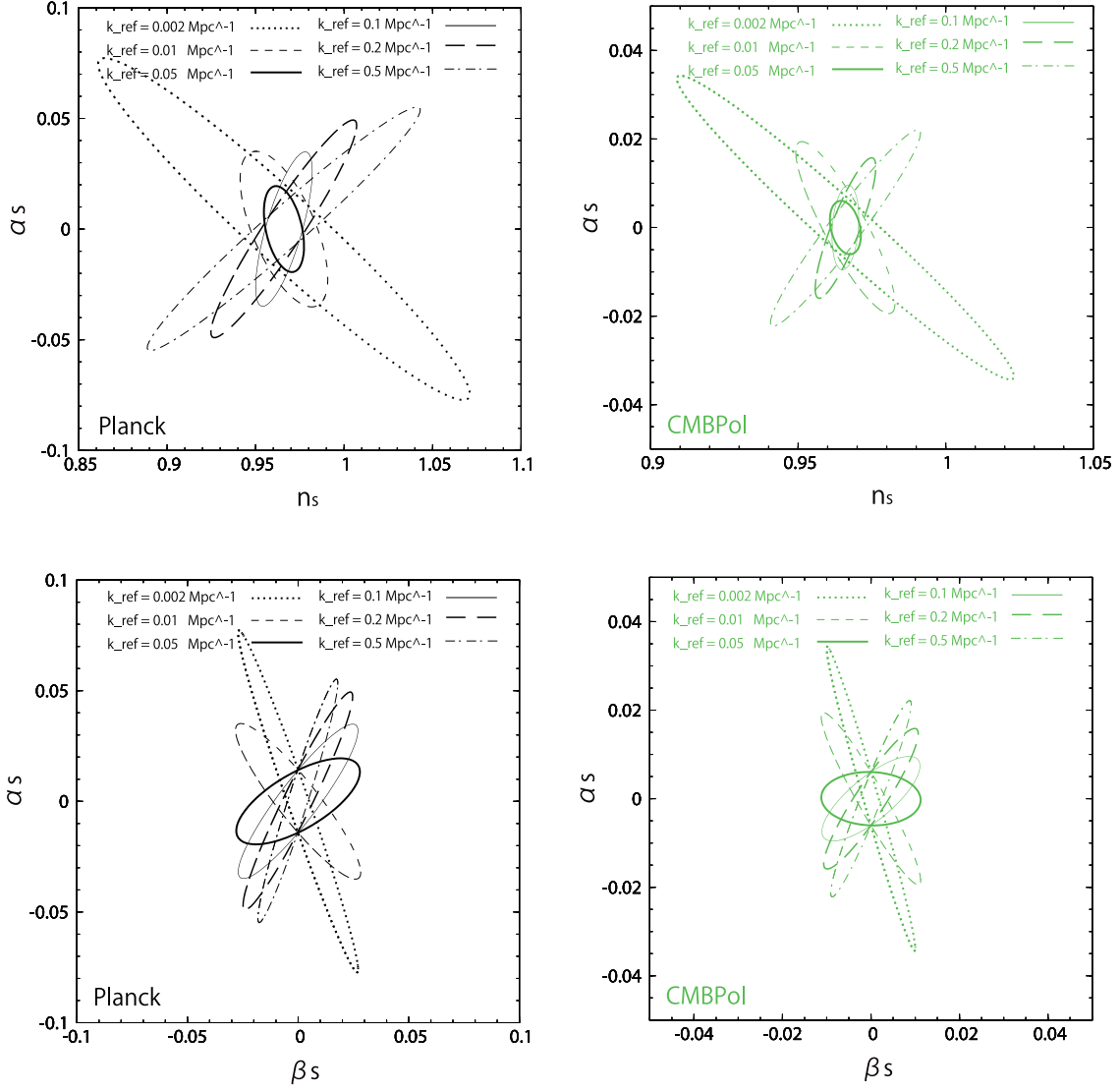


Figure 4: Projected constraints in the n_s - α_s (top panels) and α_s - β_s (bottom panels) planes for several values of k_{ref} from Planck(left) and CMBPol(right).

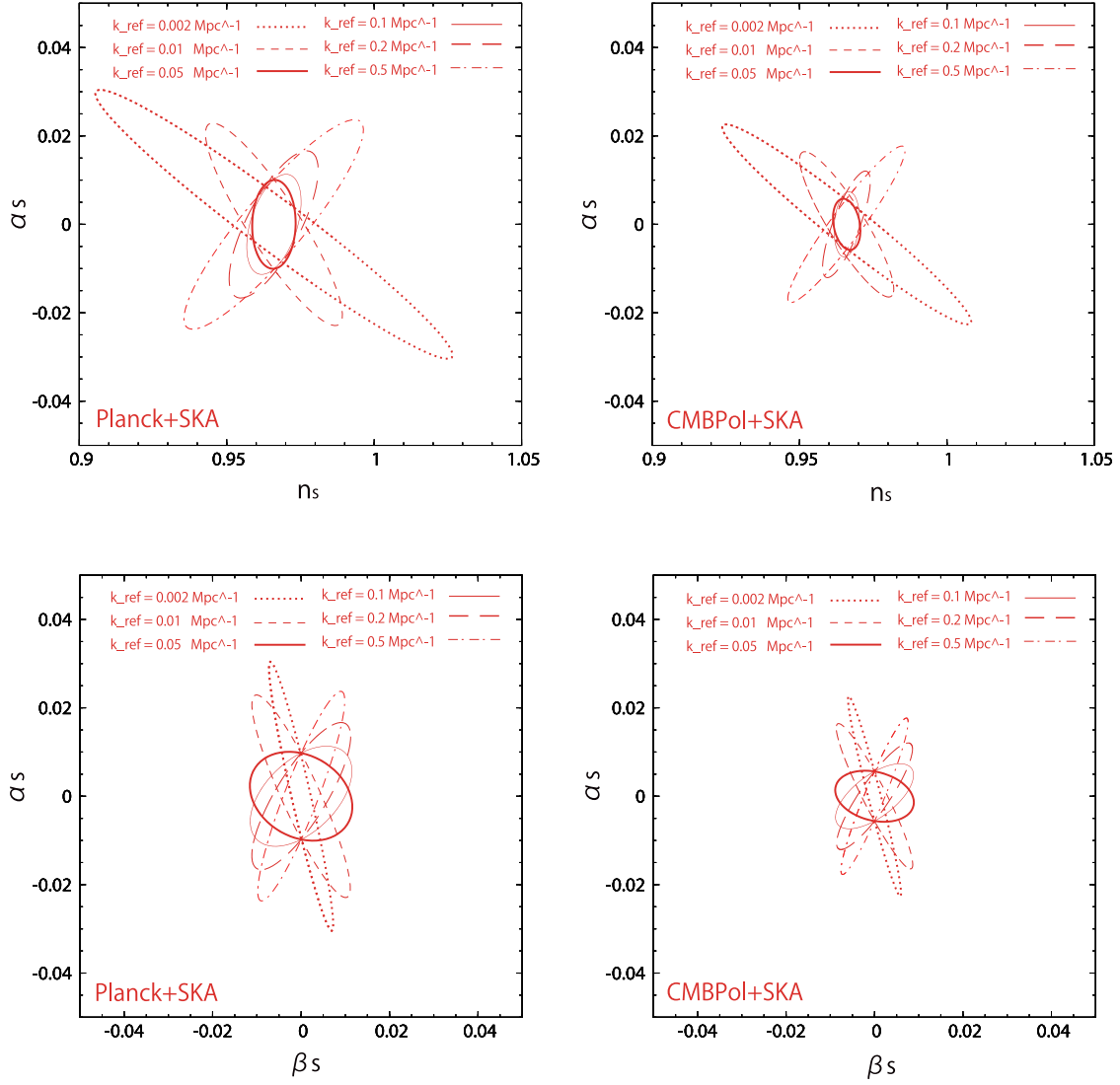


Figure 5: Same as Fig. 4, but from Planck+SKA(left) and CMBpol+SKA(right).

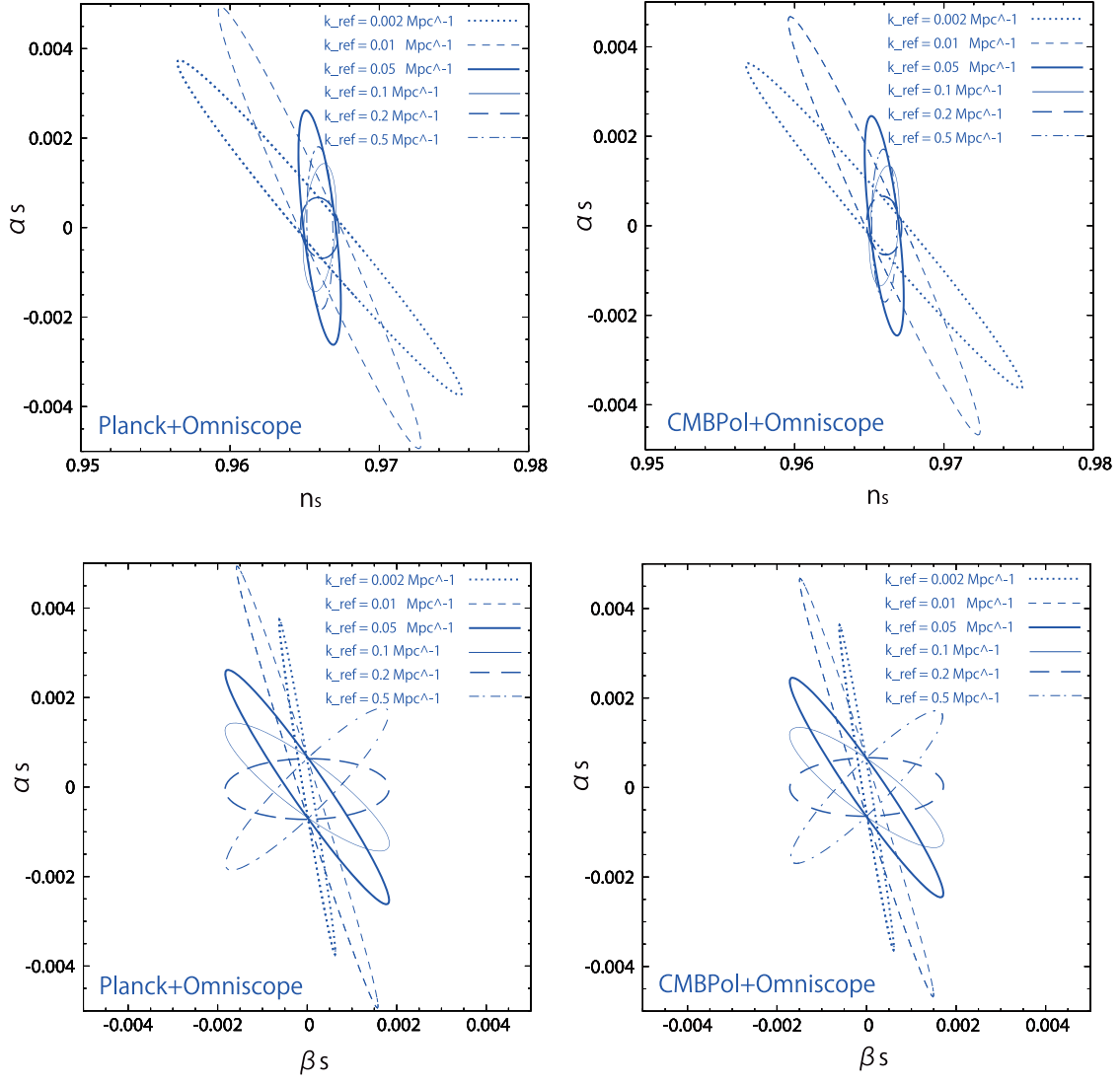


Figure 6: Same as Fig. 4, but from Planck+Omniscope (left) and CMBpol+Omniscope (right).